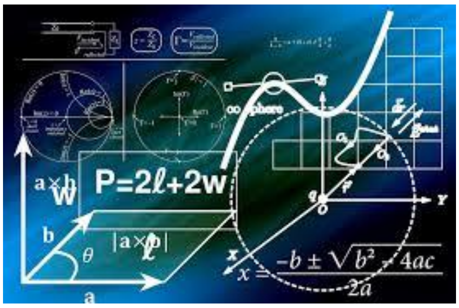
Sixth Form Preparation for Success

Welcome to Further Maths

AQA Further Mathematics specification 7367



Introduction

Further Maths builds on the knowledge and skills that you develop in the mainstream A level course. You will develop excellent problem solving skills and a deep understanding of a range of branches of mathematics. These can only be developed to the level required if underpinned by high levels of technical competency and an appreciation of mathematics 'beyond the curriculum'. The resources and tasks provided are designed to be interesting, eye opening and enjoyable as well as useful. When completed to a high standard, your work will be invaluable preparation for success ahead of your start in September.

Part I – Y11 into 12 Further Maths Specific Bridging Work
To be completed May – Sept

Remember that prizes will be awarded for 'exceptional' work that demonstrates effort above expected ! It would be a good idea to write all work that you do on lined file paper, keep it all in a file and bring it to school for your first lesson in September.

a) **Investigate places of interest** - Given the circumstances at the moment, you may not be able to physically visit the places suggested below so try the websites – many have virtual tours and mini lectures - and email their customer services with any questions; people love to hear from young people who show an interest in their line of work!

- Bletchley Park, home of the codebreakers. https://www.bletchleypark.org.uk/
- The Winton Gallery at the Science Museum, London, shows has maths has literally shaped our lives over the years. There are also links to stories of mathematicians at https://www.sciencemuseum.org.uk/see-and-do/mathematics-winton-gallery
- Mechanics link: The Spaceguard centre, part of a project tracking Near Earth Objects (NEOs). https://spaceguardcentre.com/
- Mechanics link: The UK Association for Science and Discovery Centres (ASDC). This association brings together over 60 major science engagement organisations in the UK. http://sciencecentres.org.uk/centres/weblinks.php

b) Wider reading….

- On the TV show Big Bang Theory, Schrödinger's cat is often mentioned. One great example of quantum physics' weirdness can be shown in the Schrödinger's cat thought experiment. Here is a link to an interesting TED talk on this topic. https://www.ted.com/talks/josh_samani_what_can_schrodinger_s_cat_teach_us_about_quantum_mechanics#t-9518

- Mechanics: If you were to orbit the Earth, you'd experience the feeling of free fall, not unlike what your stomach feels before a big dive on a roller coaster. With a little help from Sir Isaac Newton, Matt J. Carlson explains the basic forces acting on an astronaut and why you probably shouldn't try this one at home. https://www.ted.com/talks/dr_matt_j_carlson_free_falling_in_outer_space#t-163348
- The mathematics problem that Andrew Wiles solved had been lingering since 1637. This book is the amazing story of how this British Professor worked out a proof for the theorem that had famously bedevilled mathematicians for centuries. Fermat's Last Theorem by Simon Singh ISBN 978 – 1841157917 is highly recommended by Year 13 Further Maths students as a 'must read'.

….and for the film buffs, here are 13 maths movies based on true events that will make you think about maths and mathematicians in a different way. Visit the link for a summary of each and see as many as you can:

https://medium.com/@Alikayaspor/13-must-see-mathematics-movies-inspired-by-true-events-1beda86255cd

The Man Who Knew Infinity, Pi, A Beautiful Mind, Stand and Deliver, X + Y, Good Will Hunting, The Imitation Game, Codebreaker, A Brief History of Time, N is a Number: A portrait of Paul Erdos, Travelling Salesman, Fermat's Room, The Oxford Murders.

c) Compulsory tasks

i) Sharpening your skills

The booklet at the end of this document is aimed to keep important number and algebra skills sharp ready for September. To succeed on the course, you will need to execute basic routines and manipulate algebra fluently in all aspects of the course. Reading the notes, examples and completion of the 'Further Maths only' sections of the booklet is compulsory. Read the intro carefully as we don't expect you to complete all of the exercises work not labelled as 'Further Maths only'.

ii) Promoting a love of problem solving

The Senior Mathematical Challenge is a 90-minute, multiple-choice competition aimed at students across the UK. It encourages **mathematical reasoning**, **precision of thought**, and **fluency** in using basic mathematical techniques to solve interesting problems. The problems on the Senior Mathematical Challenge are **designed to make students think**. Most are **accessible**, yet still challenge those with more experience.
Visit https://www.ukmt.org.uk/competitions/solo/senior-mathematical-challenge where you will find out about the Senior Maths Challenge. Click on the link https://www.ukmt.org.uk/competitions/solo/senior-mathematical-challenge/archive that enables you, for FREE!, to down load past challenges, solutions and hints on how to solve the problems. As a compulsory task, you need to complete the 2019 challenge under your own 90 minute 'senior maths challenge conditions', mark it and carefully read the solutions and hints. You may choose to do more – remember *'prizes will be awarded for 'exceptional' work that demonstrates effort above expected!'* Keep any challenges you do in your file to be submitted in September.

iii) Know your stuff!

In preparation for the Statistics and Mechanics aspects of the course, you need a sound command of a number of topics from GCSE. We have set up an online class list in HegartyMaths and Mymaths , 'Y12 Preparation for Success' where you will be set a mixture lessons and quizzes which will specifically work on these important skills and prior knowledge. Contact mtwitchell@mcauley.org.uk if you need your individual login details.

d) Stretch!

The British Maths Olympiad (BMO) is a problem solving competition for the highest attainers from the UKMT Senior Maths Challenge. Its focus on ‘proof’ makes it a ‘step up’ and will get you out of your comfort zone! The BMO is a 3½-hour paper with 6 problems (the first being intended to be more accessible than the rest). Your optional ‘stretch’ task is to take a question (maybe the first question from any year you choose) from any BMO round 1 challenge and produce a model answer for it. There is no need to time your work – instead, research the question you choose, (look on forums for help?) – as you will learn more from doing one question very well rather than a rushed, incorrect solution that lacks ‘rigour’. This link, https://bmos.ukmt.org.uk/home/bmo.shtml#bmo1 , takes you straight to the page where you can download past challenges for free. This link https://bmos.ukmt.org.uk/solutions/ takes you straight to the solutions given as video presentations.

Good luck!

There now follows the Compulsory Task i) ‘Sharpen your Skills’…. that was mentioned earlier.

You will find Part II ‘Y12 Headstart!’ at the end

Compulsory Task i) Sharpening your skills

This is first part of your compulsory tasks which consolidate your mathematical skills ready for A level mathematics/further mathematics. The work in this section is not designed to teach new skills but rather to hone the techniques and skills that you hopefully already know. We recognise that students, due to various factors, will possibly not be 100% confident with these techniques and skills. Do not worry if you find this tough the first time you see it; many students find revising the ‘tough stuff’ and the ‘step up’ required from GCSE to A level a challenge regardless of their grade. You will adapt and you will get more confident the more you try.

A huge difference between A level and GCSE is the presentation of your solutions – it is not just about getting the right answer! You will be expected to show all steps in your calculations and many answers will require written sentences requiring high levels of analysis and evaluation. Please don’t look for shortcuts whilst working through this booklet. Show all steps of calculation neatly. We often ask, “Would you be proud of this work going on the wall?” Also, the new maths specifications and assessments have renewed focus and marks for ‘rigour’ ie good use of correct technical maths notation and ‘working’ to support your solutions.

The main focus of this bridging work is algebra; being able to manipulate and work with algebra fluently will give you a distinct advantage at A level. If you identify any ‘gaps’ in your knowledge then we would expect you to independently practice those skills before starting the course in September – in addition to the notes, examples and links provided, there are many excellent websites (YouTube videos are extremely helpful) where you can find the required practice. Attempt all the work and really work hard during the summer to master the techniques.

The majority of the work below is suitable as preparation for the mainstream A level course. Since you already have bridging work for mainstream maths, you may ‘pick and choose’ which questions to do from the work set in this document – maybe do every other question? – but **YOU MUST COMPLETE ALL QUESTIONS LABELLED ‘FURTHER MATHS ONLY’ !**

Read the notes and examples, watch the YouTube videos and tackle the questions. We will add the answers before half term or send them to you by email so you can check your progress!

On a final note, just because you *can* multiply out brackets, it doesn't mean that you *should*! Indeed, at this level it is often better to keep an expression in its factorised (bracketed) form.

Good luck!

Topic	**Done Exercise (✓)**	**☺ / 😐 / ☹**
1.1 - Simple algebraic expressions		
1.2 - Algebraic fractions		
1.3 - Quadratic expressions		
1.4 - Cancelling		
1.5 - Fractional and negative powers, and surds		

Algebra

Many people dislike algebra; for many it is the point at which they start switching off mathematics. But do persevere – most of it is natural enough when you think about it the right way.

1.1 Simple algebraic expressions

Some very basic things here, but they should prove helpful.

Are you fully aware that $\frac{x}{4}$ and $\frac{1}{4}x$ are the same thing?

Example 1 Find the value of a for which $\frac{8}{11}(5x-4)=\frac{8(5x-4)}{a}$ is always true.

Solution Dividing 8 by 11 and multiplying by $(5x - 4)$ is the same as multiplying 8 by $(5x - 4)$ and dividing by 11. So $a = 11$.

You do not need to multiply anything out to see this!

Remember that in algebraic fractions such as $\frac{3}{x-2}$, the line has the same effect as a bracket round the denominator. You may well find it helpful actually to *write in* the bracket: $\frac{3}{(x-2)}$.

Example 2 Solve the equation $\frac{3}{x-2}=12$.

Solution Multiply both sides by $(x - 2)$: $3 = 12(x - 2)$

Multiply out the bracket: $3 = 12x - 24$

Add 24 to both sides: $27 = 12x$

Divide by 12: $x=\frac{27}{12}=2\frac{1}{4}$.

A common mistake is to start by dividing by 3. That would give $\frac{1}{x-2}=4$ [*not* $x - 2 = 4$] and you will still have to multiply by $(x - 2)$.Don't ever be afraid to get the x-term on the *right*, as in the last line but one of the working. After all, $27 = 12x$ means just the same as $12x = 27$

Example 4 Solve the equation $\frac{3}{5}(2x+3)=\frac{7}{15}(4x-9)$

Solution Do ***not*** multiply out the brackets to get fractions – that leads to horrible numbers! Instead:

Multiply both sides by 15: $15\times\frac{3}{5}(2x+3)=15\times\frac{7}{15}(4x-9)$

Choose 15 as it gets rid of all the

Cancel down the fractions: $3\times\frac{3}{1}(2x+3)=\frac{7}{1}(4x-9)$

$$9(2x+3)=7(4x-9)$$

Now multiply out: $18x + 27 = 28x – 63$

$$90 = 10x$$

Hence the answer is $x = 9$

This makes the working very much easier. ***Please don't*** respond by saying "well, my method gets the same answer"! You want to develop your flexibility and your ability to find the easiest method if you are to do well at A Level, as well as to be able to use similar techniques in algebra instead of numbers. It's not just this example we are worried about – it's more complicated examples of a similar type.

Youtube Videos to help

Solving Linear Equations- https://www.youtube.com/watch? v=-eNM4GV-X9s

Solving Equations with algebraic fractions- https://www.youtube.com/watch?v=UkboZp0nSQs

Different Types of Equations - https://www.youtube.com/watch?v=_U2arLwb7lk

Rearranging Formulae - https://www.youtube.com/watch?v=9JRXUB2o24Y

https://www.youtube.com/watch?v=qb7qSdmJwT8

Exercise 1.1

1 Find the values of the letters p, q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \frac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \frac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\frac{60}{x+4} = 12$ (b) $\frac{35}{2x-3} = 5$ (c) $\frac{20}{6-x} = \frac{1}{2}$

3 Make cos C the subject of the formula $c^2 = a^2 + b^2 - 2ab \cos C$.

4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x + 2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations.

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations.

(a) $\frac{a}{b}(cx+d) = x+2$ (b) $\frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$

7 Simplify the following as far as possible.

(a) $\frac{a+a+a+a+a}{5}$ (b) $\frac{b+b+b+b}{b}$

(c) $\frac{c \times c \times c \times c \times c}{c}$ (d) $\frac{d \times d \times d \times d}{4}$

1.2 Algebraic Fractions

Many people have only a hazy idea of fractions. That needs improving if you want to go a long way with maths – you will need to be confident in handling fractions consisting of letters as well as numbers.

Remember, first, how to multiply a fraction by an integer. You multiply only the top *[what happens if you multiply both the top and the bottom of a fraction by the same thing?]*

Example 1 Multiply $\frac{3x}{7y}$ by 2.

Solution $3 \times 2 = 6x$, so the answer is $\frac{6x}{7y}$. (*Not* $\frac{6x}{14y}$!)

Example 2 Divide $\frac{3y^2}{4x}$ by y.

Solution $\frac{3y^2}{4x} \div y = \frac{3y^2}{4x} \times \frac{1}{y} = \frac{3y^2}{4xy} = \frac{3y}{4x}$, so the answer is $\frac{3y}{4x}$. [Don't forget to simplify.]

Double fractions, or mixtures of fractions and decimals, are always wrong.

For instance, if you want to divide $\frac{xy}{z}$ by 2, you should not say $\frac{0.5xy}{z}$ but $\frac{xy}{2z}$. This sort of thing is extremely important when it comes to rearranging and simplifying formulae.

Example 3 Make r the subject of the equation $V = \frac{1}{2}\pi r^2 h$.

Solution Multiply by 2: $2V = \pi r^2 h$

Don't "divide by $\frac{1}{2}$".

Divide by π and h: $\frac{2V}{\pi h} = r^2$

Square root both sides: $r = \sqrt{\frac{2V}{\pi h}}$.

You should *not* write the answer as $\sqrt{\frac{V}{\frac{1}{2}\pi h}}$ or $\sqrt{\frac{2V}{\pi} \div h}$, as these are fractions of fractions.

Make sure, too, that you write the answer properly. If you write $\sqrt{}2V/\pi h$ it's not at all clear that the whole expression has to be square-rooted and you will lose marks.

You will often want to combine two algebraic expressions, one of which is an algebraic fraction, into a single expression. You will no doubt remember how to add or subtract fractions, using a common denominator.

Example 4 Simplify $\frac{3}{x-1}-\frac{1}{x+1}$.

Solution Use a common denominator. [You must treat $(x - 1)$ and $(x + 1)$ as separate expressions with no common factor.]

$$\frac{3}{x-1}-\frac{1}{x+1}=\frac{3(x+1)-(x-1)}{(x-1)(x+1)}$$

$$=\frac{3x+3-x+1}{(x-1)(x+1)}=\frac{2x+4}{(x-1)(x+1)}.$$

Do use brackets, particularly on top – otherwise you are likely to forget the minus at the end of the numerator (in this example subtracting -1 gives +1).

Don't multiply out the brackets on the denominator. You will need to see if there is a factor which cancels out (although there isn't one in this case).

Example 5 Write $\frac{3}{x+1}+2$ as a single fraction.

Solution $\frac{3}{x+1}+2=\frac{3}{x+1}+\frac{2}{1}$

$$=\frac{3+2(x+1)}{x+1} \qquad =\frac{2x+5}{x+1}$$

This method often produces big simplifications when roots are involved

<u>Youtube Videos to help!</u>

Simplifying Algebraic Fractions – https://www.youtube.com/watch?v=3j2ghl_tV3Q

Adding/Subtracting Alegbraic Fractions - https://www.youtube.com/watch?v=jgGBdTL-OUw

Exercise 1.2

1 Work out the following. Answers *may* be left as improper fractions.

(a) $\frac{4}{7}\times 5$ (b) $\frac{5}{12}\times 3$ (c) $\frac{7}{9}\times 2$ (d) $\frac{4}{15}\times 3$

(e) $\frac{8}{11}\div 4$ (f) $\frac{8}{11}\div 3$ (g) $\frac{6}{7}\div 3$ (h) $\frac{6}{7}\div 5$

(i) $\frac{3x}{y}\times x$ (j) $\frac{3x}{y^2}\times y$ (k) $\frac{5x^3}{4y}\div x$ (l) $\frac{5x^2}{6y}\div y$

(m) $\frac{5x^3}{2y}\times 3x$ (n) $\frac{3y^4}{4x^2z}\times 2x$ (o) $\frac{6x^2y^3}{5z}\div 2xy$ (p) $\frac{5a^2}{6x^3z^2}\div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V = \frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u+v) = tx$ (d) $W = \frac{2}{3}\pi x^2 h$

3 Write as single fractions.

(a) $\frac{2}{x-1}+\frac{1}{x+3}$ (b) $\frac{2}{x-3}-\frac{1}{x+2}$ (c) $\frac{1}{2x-1}-\frac{1}{3x+2}$ (d) $\frac{3}{x+2}+1$ (e) $2-\frac{1}{x-1}$ (f) $\frac{2x}{x+1}-3$ (g) $\frac{3}{4(2x-1)}-\frac{1}{4x^2-1}$

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4* Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}}+\sqrt{x}$ (b) $\frac{2x}{\sqrt{x+3}}+\sqrt{x+3}$ (c) $\frac{x}{\sqrt[3]{x-2}}+\sqrt[3]{(x-2)^2}$

1.3 Quadratic Expressions

You will no doubt have done much on these for GCSE. But they are so prominent at A Level that it is essential to make sure that you are never going to fall into any traps.

First, a reminder that (a) $(x + 3)^2$ is ***not*** equal to $x^2 + 9$

(b) $\sqrt{x^2 + y^2}$ is ***not*** equal to $x + y$.

If you always remember that "square" means "multiply by itself" you will remember that

$(x+3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$.

A related process is to write a quadratic expression such as $x^2 + 6x + 11$ in the form $(x+a)^2 + b$. This is called ***completing the square***. Completing the square for quadratic expressions in which the coefficient of x^2 is 1 is very easy. The number a inside the brackets is always half of the coefficient of x.

Example 1 Write $x^2 + 6x + 4$ in the form $(x + a)^2 + b$.

Solution $x^2 + 6x + 4 = (x + 3)^2 – 9 + 4$

$= (x + 3)^2 – 5$.

This version immediately gives us several useful pieces of information. For instance, we now know a lot about the graph of $y = x^2 + 6x + 4$:

- It is a translation of the graph of $y = x^2$ by 3 units to the left and 5 units down
- Its line of symmetry is $x = –3$
- Its lowest point or vertex is at $(–3, –5)$

And we can solve the equation $x^2 + 6x + 4 = 0$ *exactly* without having to use the quadratic equation formula, to locate the roots of the function:

$x^2 + 6x + 4 = 0$

$\Rightarrow$ $(x + 3)^2 – 5 = 0$

$\Rightarrow$ $(x + 3)^2 = 5$

$\Rightarrow$ $x = –3 \pm \sqrt{5}$ [don't forget that there are two possibilities!]

<u>Youtube Videos to help!</u>

Completing the Square - https://www.youtube.com/watch?v=V65M4xNkCDs

Factorising Quadratics - https://www.youtube.com/watch?v=CLkoODzshoU

The difference of two squares - https://www.youtube.com/watch?v=Xx4kPp3SnzY

Exercise 1.3

1 Write without brackets.

(a) $(x + 5)^2$ (b) $(3x - 2)^2$ (c) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2+4}$ (b) $\sqrt{x^2-4x+4}$ (c) $\sqrt{x^2-1}$

(d) $\sqrt{x^2+9x}$ (e) $\sqrt{x^2-y^2}$ (f) $\sqrt{x^2+2xy+y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 - 5x - 6$

5 Factorise as fully as possible.

(a) $x^2 - 25$ (b) $4x^2 - 36$ (c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$ (e) $3x^2 - 5x + 2$ (f) $6x^2 - 5x - 6$

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6* Multiply out and simplify.

(a) $\left(x+\frac{1}{x}\right)^2$ (b) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ (c) $\left(x+\frac{2}{x}\right)\left(x-\frac{3}{x}\right)$

1.4 Cancelling

The word "cancel" is a very dangerous one. It means two different things, one safe enough and the other very likely to lead you astray.

You can cancel *like terms* when they are added or subtracted.

Example 1 Simplify $(x^2 - 3xy) + (3xy - y^2)$.

Solution $(x^2 - 3xy) + (3xy - y^2) = x^2 - \cancel{3xy} + \cancel{3xy} - y^2 = x^2 - y^2$.

The "$3xy$" terms have "cancelled out". This is safe enough.

It is also usual to talk about "cancelling down a fraction". Thus $\frac{10}{15} = \frac{2}{3}$. However, this tends to be very dangerous with anything other than the most straightforward numerical fractions. Consider, for instance, a fraction such as $\dfrac{x^2 + 2xy}{xy + 2y^2}$. If you try to "cancel" this, you're almost certain not to get the right answer, which is in fact $\dfrac{x}{y}$ (as we will see in Example 2, below).

Example 2 Simplify $\dfrac{x^2 + 2xy}{xy + 2y^2}$.

Solution Factorise the top as $x(x + 2y)$ and the bottom as $y(x + 2y)$:

$$\frac{x^2 + 2xy}{xy + 2y^2} = \frac{x(x + 2y)}{y(x + 2y)}$$

Now it is clear that both the top and the bottom have a factor of $(x + 2y)$.

So this can be divided out to give the answer of $\dfrac{x}{y}$.

Don't "cancel down". Factorise if you can; divide all the top and all the bottom.

Try instead to use the word "divide". What happens when you "cancel down" $\frac{10}{15}$ is that you *divide numerator and denominator* by 5. If you can divide both the numerator and denominator of a fraction by the same thing, this is a correct thing to do and you will get a simplified answer.

Taking out factors

I am sure you know that $7x^2 + 12x^3$ can be factorised as $x^2(7 + 12x)$.

You should be prepared to factorise an expression such as $7(x + 2)^2 + 12(x + 2)^3$ in the same way.

Example 3 Factorise $7(x + 2)^2 + 12(x + 2)^3$

Solution $7(x + 2)^2 + 12(x + 2)^3 = (x + 2)^2(7 + 12(x + 2))$

$= (x + 2)^2(12x + 31)$.

The only differences between this and $7x^2 + 12x^3$ are that the common factor is $(x + 2)^2$ and not x^2; and that the other factor, here $(7 + 12(x + 2))$, can be simplified.

If you multiply out the brackets you will get a cubic and you will have great difficulty in factorising that. ***Don't multiply out brackets if you can help it!***

Youtube Videos to help!

Simplifying algebraic fractions - https://www.youtube.com/watch?v=tlKN8NNNxdI

Simplifying complex fractions 1 - https://www.youtube.com/watch?v=qcGQIMRCvsM

Simplifying complex fractions 2 - https://www.youtube.com/watch?v=4p2FN3ib7is

The last 2 videos here are quite complex – these are particularly advisable for students who wish to undertake further mathematics.

Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\dfrac{4+6x}{2}$ (d) $\dfrac{4\times 6x}{2}$ (e) $\dfrac{3x+xy}{x}$

(f) $\dfrac{4x+9y}{2x+3y}$ (g) $\dfrac{4x+6y}{6x+9y}$ (h) $\dfrac{5xy+6y^2}{10x+12y}$

(i) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ (j) $\dfrac{x-3}{3-x}$ (k) $\dfrac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b} = \dfrac{py}{qz}$ (b) $\dfrac{3\pi ax}{b} = \dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab} \div \tfrac{1}{3}\pi r^3$ (b) $\dfrac{2\pi h^2}{rb} \div \tfrac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c)* $\tfrac{1}{2}k(k+1)+(k+1)$ **(d)*** $\tfrac{1}{6}k(k+1)(2k+1)+(k+1)^2$

5 Simplify as far as possible.

(a) $\dfrac{x^2+6x+8}{x^2-x-6}$ (b) $\dfrac{3x^2-2x-8}{x^2-4}$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

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(e)* $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}{x^2}$ **(f)*** $\dfrac{-\dfrac{x}{2\sqrt{1-x}} + \sqrt{1-x}}{x^2}$

1.6 Fractional and negative powers, and surds

This may seem a rather difficult and even pointless topic when you meet it at GCSE, but you will soon see that it is extremely useful at A Level, and you need to be confident with it.

Negative powers give ***reciprocals*** (1 over the power).

Fractional powers give ***roots*** (such as $\sqrt[3]{x}$).

$\mathbf{x^0 = 1}$ for any x (apart from 0^0 which is undefined).

Examples 1 (a) $\frac{1}{x^3} = x^{-3}$ (b) $\sqrt[3]{x} = x^{\frac{1}{3}}$ (c) $\pi^0 = 1$

(d) $\sqrt[4]{x^7} = x^{7/4}$. The easiest way of seeing this is to write it as $(x^7)^{\frac{1}{4}}$

You will make most use of the rules of ***surds*** when checking your answers! An answer that you give as $\frac{6}{\sqrt{3}}$ will probably be given in the book as $2\sqrt{3}$, and $\frac{2}{3-\sqrt{7}}$ as $3+\sqrt{7}$. Before worrying why you have got these wrong, you should check whether they are equivalent!

Examples 2

Indeed, they are, as we use a technique called 'rationalising the denominator'

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

and

$$\frac{2}{3-\sqrt{7}} = \frac{2}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{2(3+\sqrt{7})}{3^2-(\sqrt{7})^2} = \frac{2(3+\sqrt{7})}{9-7} = 3+\sqrt{7}.$$

The first of these processes is usually signalled by the instruction "write in surd form" and the second by "rationalise the denominator".

Remember also that to put a square root in surd form you take out the *biggest* square factor you can. Thus $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ (noting that you should take out $\sqrt{16}$ and not $\sqrt{4}$).

<u>Youtube Videos to help!</u>

Negative Powers – Exam Solutions - https://www.youtube.com/watch?v=SW9nb-13V6E

Fractional Powers – Exam Solutions - https://www.youtube.com/watch?v=_fadg_VjBMc

Exercise 1.6

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x+\frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x+\frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Further Maths Only

7* Simplify $\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{100}+\sqrt{99}}$

Part II - Year 12 Head Start! for completion June – September

Course Specification

You can find the specification to the Further Maths course on the link below. Please note that, unlike other A levels, you will be assessed in Further Maths at 'AS level' at the end of Year 12. The course is split into 3 sections. Firstly the 'Pure maths' module covers 2/3 of the course and is assessed in the Paper 1 and Paper 2 exams. The other two are 'Applied maths' modules and we the option to cover any two from Mechanics, Statistics and Discrete. At McAuley, we cover Mechanics and Statistics as these build on what you learn in mainstream A level maths and are more useful for university applications. Together these represent 1/3 of the course and are assessed in Paper 3.

https://filestore.aqa.org.uk/resources/mathematics/specifications/AQA-7367-SP-2017.PDF

Further Maths online textbook for the course

Mr Shenton will send you an access code for each of the three books you will start with in Year 12 Further Maths along with instructions on how to register them and which topics to be looking at between June and September. These books cover the compulsory content for AS and A2 of A Level Further Maths. It includes explanations about each topic, lots of worked examples, exercises, past exam questions, mock tests, answers and many thorough model answers to support independent study.

Eager to get started before September?

Mr Hegarty has now added some A Level content onto his site which you all have an account for and no doubt have used regularly in recent years. We would normally be cautious about students studying A Level content before it is taught but, for Further Maths students, I believe we should be additionally ambitious! Please see below a list of topics and quiz tasks that we recommend you research and complete. I have labelled which module each task relates to (Pure, Mechanics or Statistics), topic name and quiz number. Note: a lot of content from the mainstream maths course is needed in Further Maths.

Differentiation (mainstream AS/ A Level Maths content)

Quiz 903- 917 Pure

Optional Extension - Quiz 918 Mechanics (you would normally have studied Mechanics topics before starting this so may be one to avoid!)

Sequences and Series (mainstream AS/ A Level Maths content)

Quiz 919 – 926 Pure

Matrices (AS/ A Level Further Maths content)

Quiz 928- 940 Pure

Note: Linear programming is for the Discrete Maths module of which do not opt to take at McAuley. Therefore, you do not need to complete Quiz 941 - 943

The Mr Hayes Challenge

For those of you who do not know Mr Hayes, he is a teacher at McAuley who specialises and teaches only A Level and Further Maths groups. Please complete a challenge he has sent you below:

AS Further Mathematics – Pure Mathematics

Task 1

By means of three fully worked examples, show how to solve the following quadratic equations by the method indicated.

- $6x^2 - 7x - 5 = 0$ by factorising, giving your answers as fractions.
- $x^2 + 8x - 11 = 0$ by completing the square, giving your answers in surd form.
- $5x^2 - 12x + 6 = 0$ by using the 'formula', giving your answers correct to 4SF.

For the second equation find the sum of your roots (solutions) and the product of your roots. What do you notice? Does this also work for the other two equations?

Task 2 (Extension)

A cubic equation has the general form $ax^3 + bx^2 + cx + d = 0$, where a, b, c and d are constants (usually real numbers). Research Cardano's method for solving a cubic equation and illustrate how it works for the equation $6x^3 + 23x^2 + 11x - 12 = 0$.

There is a lot of information to digest here and a lot of work to do, some of it quite difficult without the guidance of a teacher so please do contact one of the Further Maths team at mtwitchell@mcauley.org.uk or rshenton@mcauley.org.uk or rhayes@mcauley.org.uk if you have any questions.

We very much look forward to working with you in A level Further Maths next year!